Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. Make your work correct, complete and coherent.

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 4, September 29, 2022

A projectile is fired at a speed of 840 m/sec at an angle of 60 degrees above the ground. The only force acting on the projectile is gravity (9.8 m/sec^2 toward the ground). How long will it take until the projectile is 21,000 m down range?

Answer: Let $\overrightarrow{\boldsymbol{r}}(t)$ be the position vector of the projectile at time t. We call the point where the projectile is fired the origin. We are told that

$$\begin{aligned}
\overrightarrow{\boldsymbol{r}}''(t) &= -9.8 \\
\overrightarrow{\boldsymbol{r}}'(0) &= 840\cos\tfrac{\pi}{6}\overrightarrow{\boldsymbol{i}} + 840\sin\tfrac{\pi}{6}\overrightarrow{\boldsymbol{j}} \\
&= 840(\tfrac{1}{2})\overrightarrow{\boldsymbol{i}} + 840(\tfrac{\sqrt{3}}{2})\overrightarrow{\boldsymbol{j}} \\
&= 420\overrightarrow{\boldsymbol{i}} + 420\sqrt{3}\overrightarrow{\boldsymbol{j}} \\
\overrightarrow{\boldsymbol{r}}(0) &= 0\overrightarrow{\boldsymbol{i}} + 0\overrightarrow{\boldsymbol{j}}.
\end{aligned}$$

We integrate $\overrightarrow{\boldsymbol{r}}''(t)$ to see that

$$\overrightarrow{\boldsymbol{r}}'(t) = -9.8t\overrightarrow{\boldsymbol{j}} + \overrightarrow{c}_1$$

for some constant vector $\overrightarrow{c}_1$. Plug in $t = 0$ to learn

$$420\overrightarrow{\boldsymbol{i}} + 420\sqrt{3}\overrightarrow{\boldsymbol{j}} = \overrightarrow{\boldsymbol{r}}'(0) = -9.8(0) + \overrightarrow{c}_1.$$

Thus $420\overrightarrow{\boldsymbol{i}} + 420\sqrt{3}\overrightarrow{\boldsymbol{j}} = \overrightarrow{c}_1$ and

$$\overrightarrow{\boldsymbol{r}}'(t) = 420\overrightarrow{\boldsymbol{i}} + (420\sqrt{3} - 9.8t)\overrightarrow{\boldsymbol{j}}.$$

Integrate again to learn

$$\overrightarrow{\boldsymbol{r}}(t) = 420t\overrightarrow{\boldsymbol{i}} + (420\sqrt{3}t - 4.9t^2)\overrightarrow{\boldsymbol{j}} + \overrightarrow{c}_2,$$

for some constant vector $\overrightarrow{c}_2$. Plug in $t = 0$ to learn

$$0\overrightarrow{\boldsymbol{i}} + 0\overrightarrow{\boldsymbol{j}} = \overrightarrow{\boldsymbol{r}}(0) = \overrightarrow{c}_2.$$

Thus, $\overrightarrow{c}_2 = 0$ and

$$\overrightarrow{\boldsymbol{r}}(t) = 420t\overrightarrow{\boldsymbol{i}} + (420\sqrt{3}t - 4.9t^2)\overrightarrow{\boldsymbol{j}}.$$

The projectile is 21,000 m down range when the $\vec{\boldsymbol{i}}$ component of $\vec{\boldsymbol{r}}(t)$ is equal to $21,000$. That is, when

$$420t = 21,000$$

or

$$t = \tfrac{21000}{420} = \tfrac{21\cdot 100}{21\cdot 2} = \tfrac{100}{2} = 50.$$

It takes $\boxed{\text{50 seconds}}$ for the projectile to get 21,000 m down range.