Math 241, Final Exam Spring, 2021

Write everything on the blank paper that you brought. There should be nothing on your desk except this exam, the blank paper that you brought, and a pen or pencil. When you are finished, send a picture of your solutions to

kustin@math.sc.edu

ALSO, LEAVE A PHYSICAL COPY OF YOUR SOLUTIONS WITH ME. Fold your solutions in half and write your name on the outside.

The exam is worth 100 points. There are 9 problems. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.
IF YOU LEAVE THE ROOM DURING THE EXAM, PLEASE LEAVE YOUR PHONE WITH ME BEFORE YOU GO.

(1) (11 points) Find the absolute extreme points of the function $f(x,y) = x^2 + y^2 - x - y + 1$ defined on the region $\{(x,y) \mid x^2 + y^2 \leq 1\}$.

(2) (11 points) Find the local extreme points and saddle points of the function $f(x,y) = \frac{1}{3}x^3 - 3x^2 + \frac{y^2}{4} + xy + 13x - y + 2$.

(3) (11 points) Put $3x^2+2y^2-12x+4y+9 = 0$ in the form $a(x-x_0)^2+b(y-y_0)^2 = c$, where a, b, c, x_0, and y_0 are numbers.

(4) (12 points) Let $f(x,y) = x^2 - y^2$.

(a) Draw and label a few level sets $f(x,y) = c$. Be sure to draw the level set that passes through the point $(1,2)$.

(b) Compute $\overrightarrow{\nabla} f|_{(1,2)}$.

(c) Draw your answer to (b) on your answer to (a). Be sure to put the tail of $\overrightarrow{\nabla} f|_{(1,2)}$ on $(1,2)$.

(5) (11 points) Graph, describe, and name the set of all points in 3-space which satisfy $z^2 = x^2 + y^2$.

(6) (11 points) Find the equation of the plane that contains the two lines

$$\begin{cases} x = 2t + 3 \\ y = 3t + 4 \\ z = 4t + 5 \end{cases} \quad \text{and} \quad \begin{cases} x = 2t \\ y = 3t + 1 \\ z = 4t - 2. \end{cases}$$

(7) (11 points) Find the length of the curve parameterized by

$$\overrightarrow{\boldsymbol{r}}(t) = t^2 \overrightarrow{\boldsymbol{i}} + \tfrac{8}{3} t^{3/2} \overrightarrow{\boldsymbol{j}} + 4t \overrightarrow{\boldsymbol{k}}$$

for $0 \leq t \leq 4$.

(8) (11 points) Find the area of the region between $x + y = 2$ and $y = x^2$.

(9) (11 points) Find the volume of the solid between $z = 4 - x^2 - y^2$ and $z = x^2 + y^2 - 4$.