Math 241, Final Exam, Fall, 2022

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 100 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

No Calculators, Cell phones, computers, notes, etc.

(1) Find the point on $x + 2y + 3z = 6$ that is closest to $(-3, 9, 11)$.

(2) Consider the set of points in three-space which satisfy both equations:

$$x^2 + y^2 = z^2 \quad \text{and} \quad x = 1.$$

Name, graph, and describe this set. Is the set a finite set of points, or a curve, or a surface, or a solid?

(3) An object travels in three space. The position vector of the object at time t is $\vec{\boldsymbol{r}}(t)$. Suppose that $\vec{\boldsymbol{r}}''(t) = e^t\,\vec{\boldsymbol{i}} + e^{2t}\,\vec{\boldsymbol{j}}$, $\vec{\boldsymbol{r}}'(0) = \vec{\boldsymbol{i}} + \frac{3}{2}\,\vec{\boldsymbol{j}}$, and $\vec{\boldsymbol{r}}(0) = 2\,\vec{\boldsymbol{i}} + (\frac{1}{4} - \ln 3)\,\vec{\boldsymbol{j}}$. What is the y-coordinate of the object when the x-coordinate is 4?

(4) Consider the function $f(x, y) = x - y^2$ and the point $P = (5, -2)$.

(a) Draw the level set $f(x, y) = c$ which contains the point P.

(b) Calculate $\vec{\nabla} f|_P$.

(c) Draw $\vec{\nabla} f|_P$ on your answer to (4a) with the tail on P.

(d) Calculate the directional derivative of the function f at the point P in the direction of the vector $\vec{\boldsymbol{v}} = \vec{\boldsymbol{i}} + 2\,\vec{\boldsymbol{j}}$

(5) Find all local maximum points, local minimum points, and saddle points of $f(x, y) = 7x - 8y + 2xy - x^2 + y^3$.

(6) Find the absolute minimum and absolute maximum of

$$f(x, y) = 18x^2 + 4y^2 - y^2x - 2$$

on the triangle with vertices $(-1, -1)$, $(5, -1)$ and $(5, 17)$.

PLEASE LOOK ON THE OTHER SIDE.

(7) Find the area of the region between $y = x$ and $y^2 + x = 2$. (You must draw a meaningful picture.)

(8) Let $\mathcal{S}$ be a solid. The base of $\mathcal{S}$ is the region in the xy-plane inside $x^2 + y^2 = 1$ and the top of $\mathcal{S}$ is $z = e^{x^2+y^2}$. Find the volume of $\mathcal{S}$. (You must draw a meaningful picture.)

(9) Find the volume of the solid between $z = 2 - x^2 - y^2$ and $z = x^2 + y^2 - 2$. (You must draw a meaningful picture.)

(10) Find the volume of the solid between $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{6 - x^2 - y^2}$. (You must draw a meaningful picture.)