Math 241, Final Exam , Fall, 2018

Write everything on the blank paper provided. **YOU SHOULD KEEP THIS PIECE OF PAPER.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

No Calculators, Cell phones, computers, notes, etc.

(1) Find the equation of the plane that contains the points $(-1, 2, 2)$, $(1, 1, 1)$, and $(2, 1, 3)$. Please make sure that your answer is correct.

(2) The position vector of an object at time t is $\vec{\boldsymbol{r}}(t) = t\,\vec{\boldsymbol{i}} + t^2\,\vec{\boldsymbol{j}} + t^3\,\vec{\boldsymbol{k}}$. Find the equations of the line that is tangent to the path of this object at time $t = 2$.

(3) Find the equations of the plane tangent to $z = x^2 + y^2$ when $(x, y) = (1, 2)$.

(4) Let $f(x, y) = 2x^2y^3$, $P = (1, 2)$, $\vec{\boldsymbol{v}} = 3\,\vec{\boldsymbol{i}} + 4\,\vec{\boldsymbol{j}}$. Find the directional derivative of f at the point P in the direction of $\vec{\boldsymbol{v}}$.

(5) The position vector of an object at time t is given by

$$\vec{\boldsymbol{r}}(t) = -\sin(t)\,\vec{\boldsymbol{i}} + \cos(t)\,\vec{\boldsymbol{j}}\,.$$

- (i) Eliminate the parameter and give the path of the object as an equation that involves only x and y.
- (ii) Graph the path of the object.
- (iii) Calculate $\vec{\boldsymbol{r}}\,'(\pi/2)$ and draw $\vec{\boldsymbol{r}}\,'(\pi/2)$ with the tail of this velocity vector sitting on the position of the object at time $\pi/2$.
- (iv) Calculate $\vec{\boldsymbol{r}}\,''(\pi/2)$ and draw $\vec{\boldsymbol{r}}\,''(\pi/2)$ with the tail of this acceleration vector sitting on the position of the object at time $\pi/2$.

(6) Find the absolute maximum and minimum values of

$$f(x, y) = 2 + 2x + 4y - x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, and $y = 9 - x$.

(7) Find all local maxima, local minima, and saddle points of

$$f(x, y) = y^2 + xy + 3y + 2x + 3.$$

PLEASE TURN OVER.

(8) Find the integral of $f(x, y) = x^2 + y^2$ over the region bounded by

$$y + 1 - x = 0 \quad \text{and} \quad y^2 - 1 - x = 0.$$

(9) Compute $\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2}\, dx\, dy$.

(10) Find the volume of the region below $x^2 + y^2 + z^2 = 1$ and above $z = \sqrt{x^2 + y^2}$.