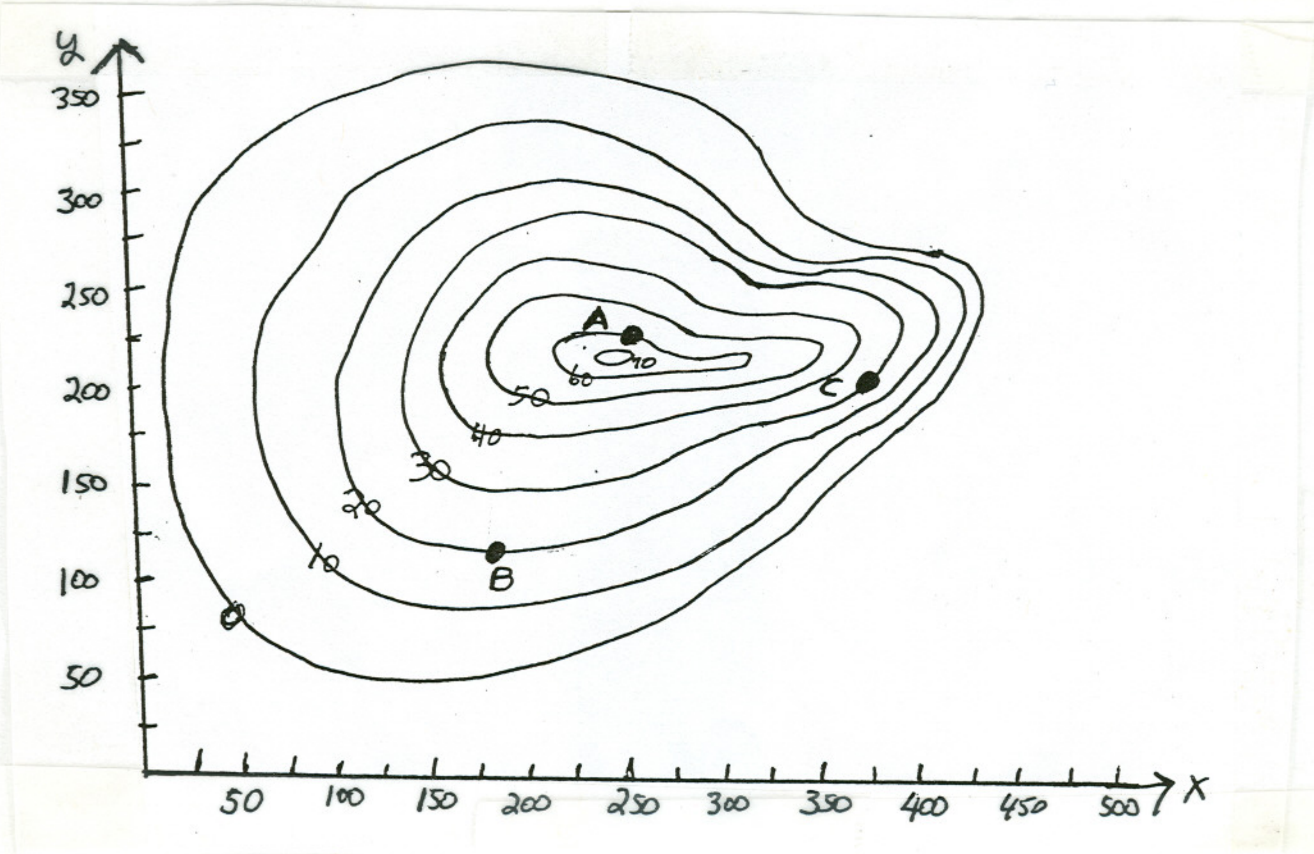
2a

Math 241, Fall 1999, exam 3

1. The picture shows the contour map for a hill 70 feet high, which we assume has the equation $z = f(x, y)$.



   (a) A raindrop landing on the hill at point A will reach the xy-plane at A', by following the path of steepest descent from A. Draw the path from A to A'.

   (b) What are the coordinates of the point A'?

   (c) Estimate f_x at the point B.

   (d) Estimate f_y at the point B.

   (e) Estimate $D_{\vec{u}} f$ at the point C, where $\vec{u} = \frac{\vec{i} + \vec{j}}{\sqrt{2}}$.

2. Let $f(x, y) = x \ln(x^2 + y^2)$. Find $\vec{\nabla} f(1, 2)$.

3. Find the directional derivative of $f(x, y) = y^2 \ln x$ at the point $(1, 2)$ in the direction of $\vec{a} = \vec{i} - \vec{j}$.

4. Find the equation of the plane tangent to the surface $z = x^3 y + 3xy^2$ at the point where $x = 2$ and $y = -2$.