MATH 241, FALL 2001, FINAL EXAM

PRINT Your Name:______________________________

Get your course grade from **TIPS/VIP** late on Tuesday or later.

There are 18 problems on 10 pages. The exam is worth a total of 150 points.

SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1. (8 points) Find the equation of the plane through $(2,1,2)$, $(2,-1,1)$, and $(4,0,0)$. **CHECK YOUR ANSWER!**

2. (8 points) Find the equations of the line through $(1,2,3)$ and $(4,6,1)$. **CHECK YOUR ANSWER!**

3. (8 points) If $f(x,y) = x\sin(xy)$, then find $\overrightarrow{\nabla} f$.

4. (8 points) Graph and describe the graph of $xz = 0$ in $3-$ space.

5. (8 points) Graph and name $x^2 + z^2 = y^2$ in $3-$ space.

6. (8 points) Graph and describe the graph of the curve $\overrightarrow{\boldsymbol{r}}(t) = \cos t\,\overrightarrow{\boldsymbol{i}} + t\,\overrightarrow{\boldsymbol{j}} + \sin t\,\overrightarrow{\boldsymbol{k}}$ in $3-$ space.

7. (8 points) What are the equations of the line tangent to the curve which is parameterized by $\overrightarrow{\boldsymbol{r}}(t) = (2t + 4t^2)\,\overrightarrow{\boldsymbol{i}} + 6t\,\overrightarrow{\boldsymbol{j}} + 2t^2\,\overrightarrow{\boldsymbol{k}}$ at $(20,12,8)$?

8. (8 points) Find the equation of the plane tangent to the surface $z = 3x^2 + y^3$ at the point where $x = 3$ and $y = -1$.

9. (14 points) The temperature of a plate at the point (x,y) is $T(x,y) = x^2 + 4y^2$.
   (a) Draw and label the level sets $T = 0$, $T = 4$, $T = 36$, $T = 64$.
   (b) A heat seeking particle always moves in the direction of the greatest increase in temperature. Place such a particle on your answer to (a) at the point $(\sqrt{3}, \frac{1}{2})$. Draw the path of the particle.
   (c) Find the equation which gives the path of the particle of part (b).

10. (8 points) Let $\overrightarrow{\boldsymbol{a}} = \overrightarrow{\boldsymbol{i}} + \overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{k}}$ and $\overrightarrow{\boldsymbol{b}} = 5\,\overrightarrow{\boldsymbol{i}} + 2\,\overrightarrow{\boldsymbol{j}} + 2\,\overrightarrow{\boldsymbol{k}}$. Find vectors $\overrightarrow{\boldsymbol{u}}$ and $\overrightarrow{\boldsymbol{v}}$ with $\overrightarrow{\boldsymbol{b}} = \overrightarrow{\boldsymbol{u}} + \overrightarrow{\boldsymbol{v}}$, $\overrightarrow{\boldsymbol{u}}$ parallel to $\overrightarrow{\boldsymbol{a}}$, and $\overrightarrow{\boldsymbol{v}}$ perpendicular to $\overrightarrow{\boldsymbol{a}}$. **CHECK YOUR ANSWER!**

11. (8 points) Find $\displaystyle\int\int_R e^{x^2+y^2}\,dA$, where R is the region inside $x^2 + y^2 = 16$.

12. (7 points) Consider the solid which is bounded by $3x + 4y + 2z = 12$ and the three coordinate planes. Find the volume of the solid. Set up the integral, **but do NOT compute the integral.**

13. (7 points) Find the volume of the region between $z = 9 - x^2 - y^2$ and the xy plane.

14. (7 points) Consider the triangle with vertices $P = (1,2,3)$, $Q = (0,2,1)$, and $R = (4,2,7)$. Find the angle of this triangle at the vertex Q.

15. (7 points) Find the directional derivative of $f(x,y) = x^3 \ln y$ at the point $(1,2)$ in the direction of $\overrightarrow{\boldsymbol{u}} = \frac{1}{\sqrt{2}}(\overrightarrow{\boldsymbol{i}} + \overrightarrow{\boldsymbol{j}})$.

16. (7 points) Suppose $\overrightarrow{\boldsymbol{r}}''(t) = \overrightarrow{\boldsymbol{i}} + e^t \overrightarrow{\boldsymbol{j}}$, $\overrightarrow{\boldsymbol{r}}'(0) = 2\overrightarrow{\boldsymbol{i}} + \overrightarrow{\boldsymbol{j}}$, and $\overrightarrow{\boldsymbol{r}}(0) = \overrightarrow{\boldsymbol{i}} + \overrightarrow{\boldsymbol{j}}$. Find $\overrightarrow{\boldsymbol{r}}(t)$.

17. (7 points) Where do the following two lines intersect? **CHECK YOUR ANSWER!**

$$\frac{x-4}{1} = \frac{y-2}{-1} = \frac{z-7}{2} \quad \text{and} \quad \frac{x+1}{-1} = \frac{y-10}{2} = \frac{z-6}{1}$$

18. (14 points) Consider the curve $\overrightarrow{\boldsymbol{r}}(t) = -2\sin t\,\overrightarrow{\boldsymbol{i}} + 3\cos t\,\overrightarrow{\boldsymbol{j}}$.
    (a) Eliminate the parameter and find an equation for this curve which involves only x and y.
    (b) Sketch the curve.
    (c) Which point on the curve corresponds to $t = \frac{\pi}{4}$.
    (d) Graph $\overrightarrow{\boldsymbol{r}}'(\frac{\pi}{4})$. Put the tail of your vector on your answer to (c).
    (e) Graph $\overrightarrow{\boldsymbol{r}}''(\frac{\pi}{4})$. Put the tail of your vector on your answer to (c).