MATH 241, FALL 2001, EXAM 3

PRINT Your Name:_____

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS! Check your answer whenever possible.** If you want to pick up your exam before Tuesday, write a short note to that effect on the top of this page and I will leave your exam outside my office door, before I go home tonight.

1. Find the equations of the line which contains $P = (1, 2, 3)$ and $Q = (7, 3, 2)$. Check your answer.

2. Find the equations of the line tangent to the curve $\overrightarrow{\boldsymbol{r}}(t) = 2t^2\,\overrightarrow{\boldsymbol{i}} + 4t^3\,\overrightarrow{\boldsymbol{j}} + 6t\,\overrightarrow{\boldsymbol{k}}$ at $t = -1$.

3. Graph and describe the graph of the curve $\overrightarrow{\boldsymbol{r}}(t) = \cos t\,\overrightarrow{\boldsymbol{i}} + t\,\overrightarrow{\boldsymbol{j}} + \sin t\,\overrightarrow{\boldsymbol{k}}$ in $3-$ space.

4. Find the length of the curve $\overrightarrow{\boldsymbol{r}}(t) = t^2\,\overrightarrow{\boldsymbol{i}} - 2t^3\,\overrightarrow{\boldsymbol{j}} + 6t^3\,\overrightarrow{\boldsymbol{k}}$ for $0 \le t \le 1$.

5. Let $f(x, y) = x^2 - y^2$. Graph and label the level sets which correspond to levels 1, 0, and -1 for this function.

6. Graph and name $z = x^2 + y^2$ in 3-space.

7. Suppose $\overrightarrow{\boldsymbol{r}}''(t) = \overrightarrow{\boldsymbol{i}} + e^{-t}\,\overrightarrow{\boldsymbol{j}}$, $\overrightarrow{\boldsymbol{r}}'(0) = 2\,\overrightarrow{\boldsymbol{i}} + \overrightarrow{\boldsymbol{j}}$, and $\overrightarrow{\boldsymbol{r}}(0) = \overrightarrow{\boldsymbol{i}} + \overrightarrow{\boldsymbol{j}}$. Find $\overrightarrow{\boldsymbol{r}}(t)$.

8. Consider the curve $\overrightarrow{\boldsymbol{r}}(t) = 2\sin t\,\overrightarrow{\boldsymbol{i}} + 3\cos t\,\overrightarrow{\boldsymbol{j}}$.
   (a) Eliminate the parameter and find an equation for this curve which involves only x and y.
   (b) Sketch the curve.
   (c) Which point on the curve corresponds to $t = -\frac{\pi}{4}$.
   (d) Graph $\overrightarrow{\boldsymbol{r}}'(-\frac{\pi}{4})$. Put the tail of your vector on your answer to (c).
   (e) Graph $\overrightarrow{\boldsymbol{r}}''(-\frac{\pi}{4})$. Put the tail of your vector on your answer to (c).

9. Find the point on $3x - 5y + 2z = 37$ which is closest to $(1, 2, 3)$.

10. Find the point of intersection of the following lines. **CHECK YOUR ANSWER!**

$$\frac{x-5}{1} = \frac{y-5}{3} = \frac{z-7}{4} \quad \text{and} \quad \frac{x+3}{2} = \frac{y+10}{3} = \frac{z+4}{1}.$$