

Math 241, Fall 2000, Exam 3

PRINT Your Name: _____

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. **CIRCLE** your answer. **NO CALCULATORS!**

- Graph and name $y = x^2 + z^2$ in 3-space.
- Graph and name $y^2 = x^2 + z^2$ in 3-space.
- Draw and label the level sets of levels $k = -1$, $k = 0$, and $k = 1$ for the function $f(x, y) = y^2 - x^2$.
- Let $f(x, y) = xe^{\frac{y}{x}}$. Find f_x and f_y .
- (15 points) Let $f(x, y) = \frac{xy + y^4}{x^2 + y^2}$.
 - Calculate the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ along $x = 0$.
 - Calculate the limit of $f(x, y)$ as $(x, y) \rightarrow (0, 0)$ along $y = x$.
 - What is $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$?
- The temperature of a metal plate at (x, y) is e^{-x-3y} degrees. A bug is walking northeast at a rate of $\sqrt{8}$ feet per minute (that is, $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = 2$). From the bug's point of view, how is the temperature changing with time as it crosses the origin?
- Find the slope of the line tangent to the curve which is the intersection of the surface $36z = 4x^2 + 9y^2$ and the plane $x = 3$ at the point $(3, 2, 2)$.
- A fly is crawling along a wire helix so that its position vector is

$$\vec{r} = 6 \cos \pi t \vec{i} + 6 \sin \pi t \vec{j} + 2t \vec{k}$$

for $0 \leq t$. At what point will the fly hit the sphere $x^2 + y^2 + z^2 = 100$ and how far did it travel in getting there (assuming it started when $t = 0$)?

- Find the equations of the line through $(4, 0, 6)$ and perpendicular to the plane $x - 5y + 2z = 10$.
- Find the equation of the plane that contains the parallel lines

$$\begin{cases} x = -2 + 2t \\ y = 1 + 4t \\ z = 2 - t \end{cases} \quad \text{and} \quad \begin{cases} x = 2 - 2t \\ y = 3 - 4t \\ z = 1 + t \end{cases}$$