

**Math 174, Fall 1998, Exam 4**

PRINT Your Name: \_\_\_\_\_

There are 12 problems on 6 pages. Four of the problems are worth 9 points. Each of the other problems is worth 8 points. CIRCLE your answers. **No Calculators.** Show your work.

1. True or False. If true, **prove** it. If false, then give a **counterexample**. If  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are functions, with  $g \circ f$  onto, then  $f$  is onto.
2. True or False. **Prove** your answer. The sets  $S = \{x \in \mathbb{R} \mid 0 < x < 1\}$  and  $U = \{x \in \mathbb{R} \mid 0 < x < 2\}$  have the same cardinality.
3. A computer programming team has 14 members: 8 of the members are women and 6 of the members are men. How many ways can a group of 7 be chosen to work on a project if at most 3 women are in the group?
4. Find the sum  $2 + 2^2 + 2^3 + 2^4 + 2^5 + \cdots + 2^{26}$ . (Your answer should not contain any dots or any summation signs.)
5. Prove  $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$ .
6. (9 points) A coin is tossed 10 times. What is the probability that exactly 5 of the tosses will land as heads?
7. (9 points) Each license plate consists of 3 digits followed by 3 letters. How many license plates are possible?
8. (9 points) How many positive integers less than 1000 have no common factors with 6?
9. (9 points) A group of 8 people attend the movies together. How many ways can they be seated in a row if two of the people (John and Mary) insist on sitting next to one another?
10. How many 5-tuples are there of the form  $(h, i, j, k, m)$  with  $1 \leq m \leq k \leq j \leq i \leq h \leq 10$ ?
11. Prove
$$\binom{5}{0} + \binom{6}{1} + \binom{7}{2} + \cdots + \binom{5+n}{n} = \binom{6+n}{n}$$
for all integers  $n$  with  $0 \leq n$ .
12. What is the coefficient of  $x^4$  in  $(3x + 2)^9$ ?