Math 174, Fall 1998, Exam 3

PRINT Your Name:⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

There are 10 problems on 4 pages. Each problem is worth 10 points.
$\boxed{CIRCLE}$ your answers. **No Calculators.** Show your work.

1. Three officers — a president, a treasurer, and a secretary — are to be chosen from among four people: Ann, Bob, Cyd, and Dan. Suppose that Bob is not qualified to be treasurer and Cyd's other commitments make it impossible for her to be secretary. How many ways can the officers be chosen?

2. Find an explicit formula for the sequence whose first few terms are $a_1 = 2$, $a_2 = -6$, $a_3 = 12$, $a_4 = -20$, $a_5 = 30$, $a_6 = -42$, $a_7 = 56$.

3. Find the sum $2 + 4 + 6 + 8 + 10 + 12 + \cdots + 1000$.

4. In the world series the first team to win four games in a row wins the series. How many ways can a world series be played if no team wins two games in a row?

5. True or False. If true, **prove** it. If false, then give a **counterexample**. If n is an integer with $1 \le n$, then

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}.$$

6. True or False. If true, **prove** it. If false, then give a **counterexample**. If n is an integer with $1 \le n$, then

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(3n-1)}{2}.$$

7. True or False. If true, **prove** it. If false, then give a **counterexample**. If n is an integer with $n \mod 4 = 3$, then $\lceil n/4 \rceil = (n+3)/4$.

8. True or False. If true, **prove** it. If false, then give a **counterexample**. For all integers a, and b, if $a|10b$, then $a|10$ or $a|b$.

9. True or False. If true, **prove** it. If false, then give a **counterexample**. For all sets A, B, and C, if $B \subseteq C$ and $A \cap C = \emptyset$, then $A \cap B = \emptyset$.

10. True or False. If true, **prove** it. If false, then give a **counterexample**. For all sets A, B, and C, if $B \subseteq C$ and $A \cap B = \emptyset$, then $A \cap C = \emptyset$.