Math 174, Exam 4, Fall 2003

PRINT Your Name:________________________________

There are 10 problems on 4 pages. Each problem is worth 5 points. The exam is worth 50 points.

CIRCLE your answers. **No Calculators.**

WHEN YOU DO SOMETHING CLEVER, EXPLAIN YOUR WORK.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your exam outside my office door later today, you may pick it up any time between then and the next class. **Let me know if you are interested.**

I will post the solutions on my website at about 12:30 today.

You may leave the binomial coefficient $\binom{n}{r}$ in any of your answers.

1. Flip a coin ten times, what is the probabilty that the coin lands "Heads" exactly three times.

2. How many license plates are possible if every license plate consists of three letters followed by three numerical digits and no letter or digit is repeated.

3. How many solutions does the equation $y_1 + y_2 + y_3 + y_4 = 32$ have, if every y_i is an integer at least 5?

4. If $f\colon X \to Y$ and $g\colon Y \to Z$ are functions and $g \circ f\colon X \to Z$ is one-to-one, must f and g both be one-to-one? Prove or give a counterexample.

5. Find the coefficient of x^7 in $(2x+3)^{10}$.

6. Given an example of a function from $\mathbb{Z}$ to $\mathbb{Z}$ which is onto, but which is not one-to-one.

7. Simplify $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n-1} + \binom{n}{n}$. (Your answer should not have any $\ldots$ or any summation signs.)

8. Solve the recurrence relation $a_0 = 3$ and $a_k = a_{k-1} + k$ for $1 \le k$.

9. A person makes an initial deposit of \$1,000 to a bank account earning interest at a rate of 6% per year compounded monthly (so the interest earned each month is $\frac{.06}{12} = .005$), and each month she adds an additional \$100 to the account. For each nonnegative integer n, let A_n be the amount in the account at the end of n months. Find a recurrence relation relating A_n to A_{n-1}.

10. A single pair of rabbits (male and female) is born at the beginning of a year. Assume:
    (1) Rabbit pairs are not fertile during the first month of life, but there after give birth to four new male/female pairs at the end of every month;
    (2) No rabbits die.

Let r_n equal the number of pairs of rabbits alive at the end of month n. Start with $r_0 = 1$. Find r_1, r_2 and r_3. Find a recurrence relation relating r_n to earlier r_k's.