Fall 2003, Math 174, Exam 2 SOLUTIONS

PRINT Your Name:________________________________

There are 10 problems on 4 pages. Each problem is worth 5 points.

$\boxed{CIRCLE}$ your answers. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your exam outside my office door later today, you may pick it up any time between then and the next class. **Let me know if you are interested.**

I will post the solutions on my website at about 1:30 today.

1. **Write 55 in base 16.**

$55 = 3(16) + 7$, so $55 = \boxed{37_{16}}$.

2. **What is negation of $3 < x \le 7$?**

$\boxed{x \le 3 \text{ or } 7 < x}$.

3. **Compute the sum $2 + 4 + 6 + 8 + \cdots + 196 + 198 + 200$.**

The sum equals $2(\sum_{k=1}^{100} k) = 2\frac{(100)(101)}{2} = \boxed{10100}$.

4. **Is the argument**

$$\begin{array}{l} p \to q \\ q \\ \therefore p \end{array}$$

valid? Jutify your answer.

The argument is $\boxed{\text{NOT valid}}$. The argument exhibits the "converse error". If you draw a truth table, you see

p	q	$p \to q$	q	p	
T	T	T	T	T	
T	F	F	F	T	
F	T	T	T	F	★
F	F	T	F	F	

Both hypotheses are true in rows 1 and 3; however, in row 3, the conclusion fails. The argument is $\boxed{\text{NOT valid}}$.

5. **True or False. If true, prove it. If false, then give a counterexample. For all integers a, b, and c, if $a|bc$, then $a|b$ or $a|c$.**

$\boxed{\text{FALSE}}$. $6|2 \cdot 3$, but $6 \not| 2$ and $6 \not| 3$.

6. **True or False. If true, prove it. If false, then give a counterexample. For all integers a and n, if $a|n^2$, then $a|n$.**

$\boxed{\text{FALSE}}$. $4|2^2$, but $4 \not| 2$.

7. **Re-write the following statement in if—then form.**
Doing his homework regularly is a necessary condition for Jim to pass the course.
(The word "necessary" may not appear in your answer.)
If Jim does not do his homework regularly, then he will not pass the course.
or
If Jim passes the course, then he will have done his homework regularly.

8. **True or False. If true, prove it. If false, then give a counterexample. For all real numbers x, $\lceil x+2 \rceil = \lceil x \rceil + 2$.**
$\boxed{\text{TRUE}}$. Let $\lceil x \rceil = n$. So, n is an integer with $n-1 < x \le n$. So, $n+1 < x+2 \le n+2$. Thus, $\lceil x+2 \rceil = n+2 = \lceil x \rceil + 2$.

9. **Prove that n^2 has the form $3k$ or $3k+1$ for all integers n.**
Let n be an arbitrary integer. There are three cases: $n = 3\ell$, $n = 3\ell + 1$, or $n = 3\ell + 2$ for some integer ℓ.

In the first case, $n = 3\ell$, so $n^2 = 9\ell^2 = 3(3\ell^2)$, which has the form $3k$, with $k = 3\ell^2$.

In the second case, $n = 3\ell + 1$, so $n^2 = 9\ell^2 + 6\ell + 1 = 3(3\ell^2 + 2\ell) + 1$, which has the form $3k+1$, with $k = 3\ell^2 + 2\ell$.

In the third case, $n = 3\ell + 2$, so $n^2 = 9\ell^2 + 12\ell + 4 = 3(3\ell^2 + 4\ell + 1) + 1$, which has the form $3k+1$, with $k = 3\ell^2 + 4\ell + 1$.

10. **Prove that $\sqrt{5}$ is irrational.**
We employ the technique of "Proof by Contradiction".
Suppose that $\sqrt{5}$ is rational.
So, $\sqrt{5} = a/b$, where a, and b are integers with no common factors and $b \neq 0$.
Multiply both sides by b: $b\sqrt{5} = a$.
Square both sides: $b^2 5 = a^2$.
So, the prime number 5 divides into a^2. The unique factorization theorem tells us that 5 divides into a. In other words, $a = 5c$ for some integer c.
Substitute to get: $b^2 5 = 25c^2$.
Divide both sides by 5: $b^2 = 5c^2$.
So, the prime number 5 divides into b^2. The unique factorization theorem tells us that 5 divides into b.
We have now reached a contradiction. We had assumed that a and b were relatively prime. However we showed that a and b must have a common factor of 5. This is impossible. Every step was legal, except possibly our original supposition. Our supposition must be false; hence, $\sqrt{5}$ is irrational.