f

9. True or False. If true, **prove** it. If false, then give a **counterexample**. For all sets A, B, and C, $A \setminus (B \setminus C) = (A \setminus B) \setminus C$.

False $A = B = C = \{1\}$

$A \setminus (B \setminus C) = A \setminus \emptyset = \{1\}$

$(A \setminus B) \setminus C = \emptyset \setminus C = \emptyset$

10. Solve the recurrence relation $d_k = 2d_{k-1} + 3$, for all integers $k \geq 2$, $d_1 = 2$.

$d_1 = 2$

$d_2 = 2 \cdot 2 + 3$

$d_3 = 2(2 \cdot 2 + 3) + 3$

$d_4 = 2(2^3 + 2 \cdot 3 + 3) + 3$

$d_5 = 2(2^4 + 2^2 \cdot 3 + 2 \cdot 3 + 3) + 3$

$d_n = 2^n + 3(2^{n-2} + 2^{n-3} + \cdots + 2^1 + 2^0)$

$= 2^n + 3 \frac{2^{n-1} - 1}{2 - 1}$

$= 2^{n-1}(2+3) - 3$

We guess $d_n = 5(2^{n-1}) - 3$

Check using Induction

$d_1 = 5 \cdot 2^0 - 3 = 2$ ✓

If $d_n = 5 \cdot 2^{n-1} - 3$, then

$d_{n+1} = 2 \cdot d_n + 3 = 2(5 \cdot 2^{n-1} - 3) + 3$

$= 5 \cdot 2^n - 6 + 3 = 5 \cdot 2^n - 3$ ✓

11. A single pair of rabbits (male and female) is born at the beginning of a year. Let r_n equal the number of rabbit pairs alive at the end of month n, for each integer $n \geq 1$, and let $r_0 = 1$. Find a recurrence relation for $r_0, r_1, r_2, r_3, \ldots$. Assume the following conditions:
    (a) Rabbit pairs are not fertile during their first month of life, but thereafter give birth to four new male/female pairs at the end of every month.
    (b) No deaths occur during the year.

$r_0 = 1$

$r_1 = 1$

$r_2 = 1 + 4$

$r_3 = 5 + 4 \cdot 1$ $= 9$

$r_4 = 9 + 4 \cdot 5$ $= 29$

$r_n = r_{n-1} + 4r_{n-2}$ for $n \geq 1$

$r_0 = 1$