

6. Prove $\binom{6}{0} + \binom{7}{1} + \binom{8}{2} + \dots + \binom{6+n}{n} = \binom{7+n}{n}$ for all integers n with $0 \leq n$.

Proof by induction. If $n=0$, then the LHS = $\binom{6}{0}$ and the RHS = $\binom{7}{0}$; these are equal.

IH Assume $\binom{6}{0} + \binom{7}{1} + \dots + \binom{6+n}{n} = \binom{7+n}{n}$.

We must show $\binom{6}{0} + \binom{7}{1} + \dots + \binom{6+n}{n} + \binom{6+n+1}{n+1} = \binom{7+n+1}{n+1}$ *

But the LHS of * $\stackrel{\uparrow}{=} \binom{7+n}{n} + \binom{6+n}{n+1} \stackrel{\uparrow}{=} \binom{8+n}{n+1}$ and this is the RHS of *;
 use IH ↑ the identity from Pascal's triangle

the proof is complete

7. Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:

- a. Socko: Lefty killed Sharky.
 - b. Fats: Muscles didn't kill Sharky.
 - c. Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
 - d. Muscles: Lefty didn't kill Sharky.
- Who did kill Sharky?

If Lefty killed Sharky, then Socko & Fats are both telling the truth. This is not possible. Thus Lefty did not kill Sharky; so Muscles tells the truth. Thus Muscles is the only one telling the truth, so Fats is lying. ∴ Muscles did kill Sharky

8. Write the following sentence in if-then form: "A sufficient condition for Hal's team to win the championship is that it win the rest of its games".

If Hal's team wins the rest of its games, then it wins the championship.