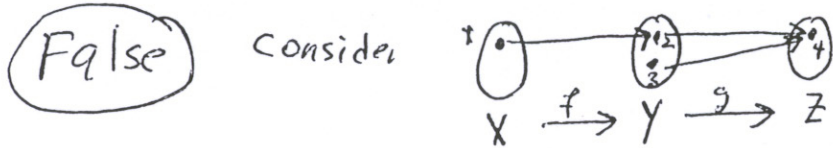# Math 174 Fall 1998 Exam 4

PRINT Your Name:_____

There are 12 problems on 6 pages. Four of the problems are worth 9 points. Each of the other problems is worth 8 points. CIRCLE your answers. **No Calculators.** Show your work.

1. True or False. If true, **prove** it. If false, then give a **counterexample**. If $f: X \to Y$ and $g: Y \to Z$ are functions, with $g \circ f$ onto, then f is onto.

(False) Consider

$$X \xrightarrow{f} Y \xrightarrow{g} Z$$

$g \circ f$ is onto but f is not onto

2. True or False. **Prove** your answer. The sets $S = \{x \in \mathbb{R} \mid 0 < x < 1\}$ and $U = \{x \in \mathbb{R} \mid 0 < x < 2\}$ have the same cardinality.

(True) Consider $f: S \longrightarrow U$ given by $f(x) = 2x$

We see that f is onto. If $y \in U$ then $0 < y < 2$ so $0 < \frac{y}{2} < 1$

so $\frac{y}{2} \in S$ and $f(\frac{y}{2}) = y$.

We see that f is 1-1: If $f(x) = f(y)$ then $2x = 2y$ so $x = y$.