

Math 174, Fall 2003, Solution to Quiz 3

Question: Prove that the square of any integer has the form $3k$ or $3k + 1$ for some integer k .

Answer: Let n be an arbitrary integer. There are three cases: $n = 3\ell$, $n = 3\ell + 1$, or $n = 3\ell + 2$ for some integer ℓ .

In the first case, $n = 3\ell$, so $n^2 = 9\ell^2 = 3(3\ell^2)$, which has the form $3k$, with $k = 3\ell^2$.

In the second case, $n = 3\ell + 1$, so $n^2 = 9\ell^2 + 6\ell + 1 = 3(3\ell^2 + 2\ell) + 1$, which has the form $3k + 1$, with $k = 3\ell^2 + 2\ell$.

In the third case, $n = 3\ell + 2$, so $n^2 = 9\ell^2 + 12\ell + 4 = 3(3\ell^2 + 4\ell + 1) + 1$, which has the form $3k + 1$, with $k = 3\ell^2 + 4\ell + 1$.