The vat shown in the accompanying figure contains water to a depth of 2 m. Find the work required to pump all the water to the top of the vat. [Use 9810 N/m^3 as the weight density of water.]

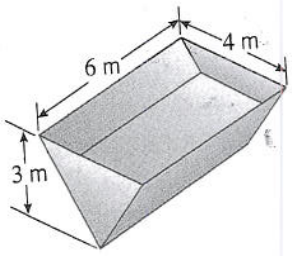

Figure Ex-15

Notice how I have oriented my axis. I put $x=0$ at the top of the vat. I will calculate the work required to lift the thin layer of water whose x-coordinate is x. I will do this for each x from $x=1$ to $x=3$. The work to lift the water at x-coordinate x:

Vol of water · Density of water · distance lifted

$= w \cdot \ell \cdot t \cdot 9810 \frac{N}{m^3} \cdot x$

$t = dx$ $\ell = 6m$ use similar triangles to see

so $\frac{\frac{w}{2}}{2} = \frac{3-x}{3}$ or $w = \frac{4}{3}(3-x)$

The total work done is $\int_1^3 \frac{4}{3}(3-x)\,6\,(9810)\,x\,dx$

$= 8(9810)\int_1^3 (3x-x^2)\,dx = 8(9810)\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_1^3$

$= 8(9810)\left(\frac{27}{2} - 9 - \frac{3}{2} + \frac{1}{3}\right) = \boxed{8(9810)\left(\frac{10}{3}\right)\text{ J.}}$

Feb. 2 Quiz