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Quiz – September 14, 2006

Find the length of the curve

$$\begin{cases} x = \frac{1}{3}t^3 \\ y = \frac{1}{2}t^2, \end{cases}$$

for $0 \leq t \leq 1$.

Answer: The arc length is equal to

$$\begin{aligned} \int_{\text{beginning}}^{\text{end}} ds &= \int_{\text{beginning}}^{\text{end}} \sqrt{dx^2 + dy^2} = \int_{\text{beginning}}^{\text{end}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \\ &= \int_0^1 \sqrt{(t^2)^2 + (t)^2} dt = \int_0^1 \sqrt{t^4 + t^2} dt = \int_0^1 t\sqrt{t^2 + 1} dt = \frac{1}{2} \frac{2}{3} (t^2 + 1)^{\frac{3}{2}} \Big|_0^1 \\ &= \frac{1}{2} \frac{2}{3} (2^{\frac{3}{2}} - 1) = \boxed{\frac{1}{3} (2^{\frac{3}{2}} - 1)} \end{aligned}$$