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**Quiz – August 27, 2004**

Let  $f(x) = 4x^2$  for  $x \leq 0$ .

(a) Find  $f^{-1}(x)$ .

(b) What is the domain of  $f^{-1}(x)$ ?

(c) Verify that  $f(f^{-1}(x)) = x$  for all  $x$  in the domain of  $f^{-1}$ .

**Answer:** Let  $y = f^{-1}(x)$ . So  $f(y) = x$ . (Notice, for future reference, that  $y$  is in the domain of  $f$ , so  $y \leq 0$ .) At any rate, we have  $4y^2 = x$  and we want to find  $y$ . Divide both sides by 4 to get  $y^2 = \frac{1}{4}x$ . Take the square root of both sides to get  $y = \pm\frac{1}{2}\sqrt{x}$ . We aren't finished because we have not found a **function** for  $f^{-1}(x)$ . However, we saw above that  $y \leq 0$ ; so  $y = -\frac{1}{2}\sqrt{x}$ . Our answer to (a) is

$f^{-1}(x) = -\frac{1}{2}\sqrt{x}$ . Our answer to (b) is  $\text{all } x \geq 0$ ; because, we can insert any number which is non-negative into  $\sqrt{x}$ . For part (c), we start with  $x \geq 0$ . We observe that

$$f(f^{-1}(x)) = f\left(-\frac{1}{2}\sqrt{x}\right) = 4\left(-\frac{1}{2}\sqrt{x}\right)^2 = 4\frac{1}{4}(\sqrt{x})^2 = x. \checkmark$$