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Quiz – March 2, 2004

Find

$$\int \frac{2x^2 + x - 8}{x^3 + 4x} dx.$$

Answer: Set

$$\frac{2x^2 + x - 8}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}.$$

Multiply both sides by $x(x^2 + 4)$ to get

$$2x^2 + x - 8 = A(x^2 + 4) + (Bx + C)x.$$

That is,

$$2x^2 + x - 8 = (A + B)x^2 + Cx + 4A.$$

Equate the corresponding coefficients to get:

$$\begin{aligned} 2 &= A + B \\ 1 &= C \\ -8 &= 4A. \end{aligned}$$

The bottom equation says $A = -2$. The middle equation says $C = 1$. The top equation says $B = 4$. We check what we have so far:

$$\frac{-2}{x} + \frac{4x + 1}{x^2 + 4} = \frac{-2(x^2 + 4) + (4x + 1)x}{x(x^2 + 4)} = \frac{2x^2 + x - 8}{x(x^2 + 4)}.\checkmark$$

The original problem is

$$\int \left(\frac{-2}{x} + \frac{4x + 1}{x^2 + 4} \right) dx = \boxed{-2 \ln |x| + 2 \ln(x^2 + 4) + \frac{1}{2} \arctan \left(\frac{x}{2} \right) + C.}$$

By the way, the derivative of $\frac{1}{2} \arctan(\frac{x}{2})$ is

$$\frac{1}{2} \frac{\frac{1}{2}}{1 + \left(\frac{x}{2}\right)^2} = \frac{1}{4\left(1 + \frac{x^2}{4}\right)} = \frac{1}{4 + x^2},$$

as expected.