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Quiz – February 12, 2004

Find

$$\int \sqrt{5 - 4x - x^2} dx.$$

Check your answer if you have time and energy.

Answer: Complete the square:

$$5 - 4x - x^2 = 5 + 4 - (x^2 + 4x + 4)$$

to see that the original problem is equal to

$$\int \sqrt{9 - (x + 2)^2} dx.$$

I find the expression $a^2 - u^2$ to be ugly in this problem, so I let $u = a \sin \theta$; that is, I let $x + 2 = 3 \sin \theta$. It follows that $dx = 3 \cos \theta d\theta$. Notice that

$$\sqrt{9 - (x + 2)^2} = \sqrt{9 - 9 \sin^2 \theta} = 3 \cos \theta.$$

The original problem is

$$\begin{aligned} 9 \int \cos^2 \theta d\theta &= \frac{9}{2} \int 1 + \cos 2\theta d\theta = \frac{9}{2} \left(\theta + \frac{\sin 2\theta}{2} \right) + C \\ &= \frac{9}{2} \left(\theta + \frac{2 \sin \theta \cos \theta}{2} \right) + C = \boxed{\frac{9}{2} \left(\arcsin \left(\frac{x + 2}{3} \right) + \frac{(x + 2)\sqrt{9 - (x + 2)^2}}{9} \right) + C.} \end{aligned}$$

Check: The derivative of the proposed answer is

$$\begin{aligned} &\frac{9}{2} \left(\frac{1}{3\sqrt{1 - \frac{(x+2)^2}{9}}} + \frac{1}{9} \left((x+2) \frac{-2(x+2)}{2\sqrt{9 - (x+2)^2}} + \sqrt{9 - (x+2)^2} \right) \right) \\ &= \frac{9}{2} \left(\frac{1}{3\sqrt{\frac{9-(x+2)^2}{9}}} + \frac{1}{9} \left(\frac{-(x+2)^2}{\sqrt{9 - (x+2)^2}} + \frac{9 - (x+2)^2}{\sqrt{9 - (x+2)^2}} \right) \right) \\ &= \frac{9}{2} \left(\frac{1}{\sqrt{9 - (x+2)^2}} + \frac{9 - 2(x+2)^2}{9\sqrt{9 - (x+2)^2}} \right) \\ &= \frac{9}{2} \left(\frac{9}{9\sqrt{9 - (x+2)^2}} + \frac{9 - 2(x+2)^2}{9\sqrt{9 - (x+2)^2}} \right) \\ &= \frac{9}{2} \left(\frac{2(9 - (x+2)^2)}{9\sqrt{9 - (x+2)^2}} \right) \\ &= \frac{9 - (x+2)^2}{\sqrt{9 - (x+2)^2}} = \sqrt{9 - (x+2)^2} = \sqrt{9 - (x^2 + 4x + 4)} = \sqrt{5 - 4x - x^2}. \checkmark \end{aligned}$$