PRINT Your Name:____________________

Quiz – November 30, 2006

Does the series

$$1-\frac{1}{2}+\frac{2}{3}-\frac{1}{3}+\frac{2}{4}-\frac{1}{4}+\frac{2}{5}-\frac{1}{5}+\dots$$

converge? Explain **very thoroughly.**

Answer: NO! We show that the sequence of partial sums does not converge. Consider the even number $2n$. The $2n^{\text{th}}$ partial sum of our series is

$$s_{2n}=1-\frac{1}{2}+\frac{2}{3}-\frac{1}{3}+\frac{2}{4}-\frac{1}{4}+\frac{2}{5}-\frac{1}{5}+\dots+\frac{2}{n}-\frac{1}{n}$$

$$=\left(1-\frac{1}{2}\right)+\left(\frac{2}{3}-\frac{1}{3}\right)+\left(\frac{2}{4}-\frac{1}{4}\right)+\left(\frac{2}{5}-\frac{1}{5}\right)+\dots+\left(\frac{2}{n}-\frac{1}{n}\right)$$

$$=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots+\frac{1}{n},$$

and this is 1 less than the n^{th} partial sum of the DIVERGENT harmonic series $\sum\limits_{k=1}^{\infty}\frac{1}{k}$. Thus, $\lim\limits_{n\to\infty}s_{2n}=+\infty$ and the series

$$1-\frac{1}{2}+\frac{2}{3}-\frac{1}{3}+\frac{2}{4}-\frac{1}{4}+\frac{2}{5}-\frac{1}{5}+\dots$$

diverges.