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Quiz – November 30, 2004

The tenth Taylor polynomial for $\sin x$ about $x = 0$ is

$$P_{10}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}.$$

Estimate the error that is introduced if $P_{10}(x)$ is used in place of $\sin x$ for $0 \le x \le \frac{\pi}{2}$. Justify your answer.

Answer: Taylor's Theorem tells us that

$$|\sin x - P_{10}(x)| = |R_{10}(x)| = \left|\frac{f^{(11)}(c)x^{11}}{11!}\right|,$$

for some c with $0 \le c \le x$, where $f(x) = \sin x$. We know that $f^{(11)}(x) = -\sin(x)$; hence, $|f^{(11)}(c)| \le 1$. It follows that

$$|\sin x - P_{10}(x)| \le \left|\frac{(\frac{\pi}{2})^{11}}{11!}\right|.$$

If you have a calculator handy, then you can calculate that $\frac{(\frac{\pi}{2})^{11}}{11!} \cong 3.6 \times 10^{-6}$. The conclusion is that if $0 \le x \le \frac{\pi}{2}$, then $P_{10}(x)$ approximates $\sin x$ with an error of at most 3.6×10^{-6}.