PRINT Your Name:________________

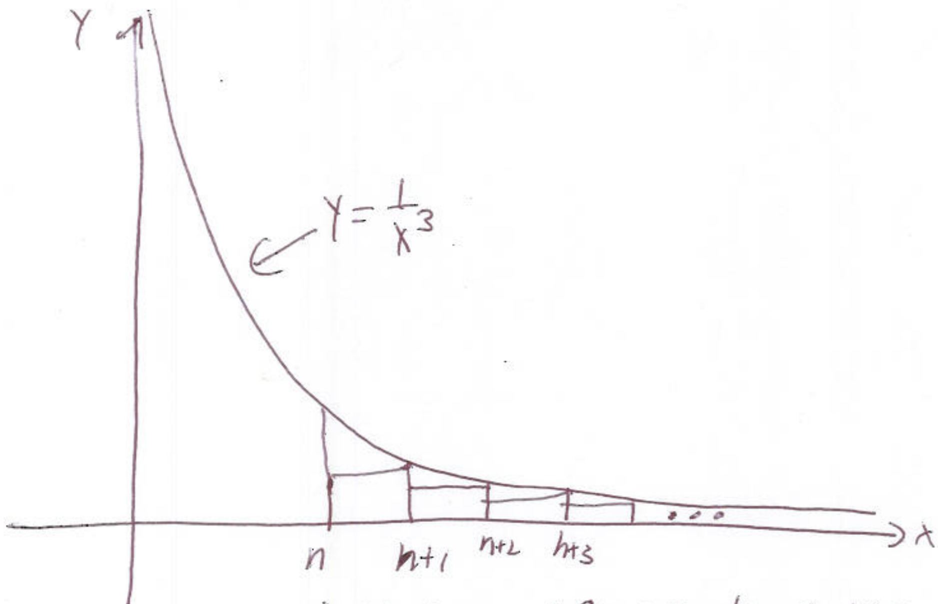
Quiz – November 15, 2006

Approximate $\sum_{k=1}^{\infty} \frac{1}{k^3}$ with an error of at most $\frac{1}{200}$. Explain **very thoroughly.**

Answer: Let $S = \sum_{k=1}^{\infty} \frac{1}{k^3}$. I approximate S by $s_n = \sum_{k=1}^{n} \frac{1}{k^3}$. It is clear that

$$|S - s_n| = \sum_{k=n+1}^{\infty} \frac{1}{k^3}.$$

Draw a picture



area inside boxes ≤ area under curve

$\sum_{k=n+1}^{\infty} \frac{1}{k^3} \le \int_n^{\infty} \frac{1}{x^3}\,dx$

to show that

$$|S - s_n| = \sum_{k=n+1}^{\infty} \frac{1}{k^3} \le \int_n^{\infty} \frac{1}{x^3} dx = \lim_{b\to\infty} \left.\frac{1}{-2x^2}\right|_n^b = \lim_{b\to\infty} \frac{1}{-2b^2} + \frac{1}{2n^2} = \frac{1}{2n^2}.$$

To make $|S - s_n| \le \frac{1}{200}$, I make $\frac{1}{2n^2} \le \frac{1}{200}$; that is, I make $100 \le n^2$, or $10 \le n$.
I conclude that

$$\boxed{\text{If } 10 \le n, \text{ then } \sum_{k=1}^{n} \frac{1}{k^3} \text{ approximates } \sum_{k=1}^{\infty} \frac{1}{k^3} \text{ with an error of at most } \frac{1}{200}.}$$