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Quiz – October 12, 2006

Does the integral $\displaystyle\int_2^\infty \frac{x}{x^5+1}dx$ converge? If so, find an upper bound on the value of the integral.

Answer: It would take a lot of effort to factor x^5+1 into linear and quadratic factors, apply the technique of partial fractions to x over the factorizattion of of x^5+1, complete the square for the quadratic factors, and do the trig substitution to the resulting expressions in order to get the exact value of $\displaystyle\int_2^\infty \frac{x}{x^5+1}dx$. But, I wasn't asked for the exact value of this integral. I was only asked if the integral is finite and if so, then what is an upper bound for the integral. Notice that

$$0 \le \frac{x}{x^5+1} \le \frac{x}{x^5} = \frac{1}{x^4}.$$

It follows that

$$\int_2^\infty \frac{x}{x^5+1}dx \le \int_2^\infty \frac{1}{x^4}dx = \lim_{b\to\infty} \frac{1}{-3x^3}\Big|_2^b = \lim_{b\to\infty} \frac{1}{-3b^3} + \frac{1}{24} = \frac{1}{24}.$$

We conclude that $\displaystyle\int_2^\infty \frac{x}{x^5+1}dx$ does converge and the value of this integral is at most $\frac{1}{24}$.