Quiz 9 October 20, 2010 – Section 9 – 10:10 – 11:00

Does the series $\sum_{n=1}^{\infty} \frac{1+4^n}{1+10^n}$ converge? **Justify your answer very thoroughly.**

Answer. Compare the given series to $\sum_{n=1}^{\infty} \frac{4^n}{10^n}$, which is a geometric series with ratio $\frac{4}{10}$, which is less than 1. Thus, $\sum_{n=1}^{\infty} \frac{4^n}{10^n}$ converges. We use the limit comparison test. Both series are series of positive numbers and

$$\lim_{n\to\infty} \frac{\frac{4^n}{10^n}}{\frac{1+4^n}{1+10^n}} = \lim_{n\to\infty} \frac{4^n}{10^n}\frac{1+10^n}{1+4^n} = \lim_{n\to\infty} \frac{\frac{1}{10^n}(1+10^n)}{\frac{1}{4^n}(1+4^n)} = \lim_{n\to\infty} \frac{\frac{1}{10^n}+1}{\frac{1}{4^n}+1} = 1.$$

We know that 1 is a number which is not zero or ∞. The limit comparison test tells us that both series converge or both series diverge. We have already seen that $\sum_{n=1}^{\infty} \frac{4^n}{10^n}$ converges. We conclude that $\sum_{n=1}^{\infty} \frac{1+4^n}{1+10^n}$ converges.