PRINT Your Name:__________________

Quiz 9 — March 16, 2012 – Section 7 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Write in complete sentences.
The quiz is worth 5 points.

Consider the series $\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$.

1. Find a closed formula for the N^{th} partial sum $s_N = \sum_{n=1}^{N}(e^{1/n} - e^{1/(n+1)})$. **Please notice that N and n are different!**
2. Does the series $\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$ converge? Explain your answer very thoroughly.
3. Find the sum of the series $\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$, if possible. Explain your answer very thoroughly.

Answer:

1. We see that

$$s_N = \sum_{n=1}^{N}(e^{1/n} - e^{1/(n+1)}) = (e^{1/1} - e^{1/2}) + (e^{1/2} - e^{1/3}) + \cdots + (e^{1/N} - e^{1/(N+1)})$$

$$= e - e^{1/(N+1)}.$$

2. and 3. We compute $\lim_{N\to\infty} s_N = \lim_{N\to\infty}(e - e^{1/(N+1)}) = e - e^0 = e - 1$. We conclude that the series $\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$ converges to $e - 1$.