PRINT Your Name:____________________

Quiz 8 — October 17, 2012 – Section 10 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Circle your answer. **Show your work.** Your work should be correct and coherent.
The quiz is worth 5 points.

Does the sequence whose n^{th} term is $a_n = (1 + \frac{2}{n})^n$ converge? If so, what is the limit of the sequence? Explain your answer very thoroughly.

Answer: We compute $\lim\limits_{n\to\infty} a_n$. The base goes to 1 and the exponent goes to infinity. The closer the base is to 1 the more the exponent tries to make the answer be infinite. This is an indeterminate form (that is, one can not tell the answer from the form of the problem alone – one must dig more deeply into to the functions that comprise our problem). We will turn the problem into a quotient and use L'hopital's rule. To do this, we will take the ln of a_n because $\ln a_n$ will turn the parts of the problem which are battling one another (namely, the exponent and the base) into a product. Routine tricks can be used to turn the product into a quotient. After we find $\lim\limits_{n\to\infty} \ln a_n$, then we will exponentiate to find $\lim\limits_{n\to\infty} a_n$. At any rate,

$$\lim_{n\to\infty} \ln a_n = \lim_{n\to\infty} n \ln\left(1 + \frac{2}{n}\right) = \lim_{n\to\infty} \frac{\ln(1+\frac{2}{n})}{\frac{1}{n}}.$$

At this point the top and the bottom both go to infinity, so we may apply L'hopital's rule to see that

$$\lim_{n\to\infty} \ln a_n = \lim_{n\to\infty} \frac{\frac{\frac{-2}{n^2}}{1+\frac{2}{n}}}{\frac{-1}{n^2}} = \lim_{n\to\infty} \frac{\frac{2}{1+\frac{2}{n}}}{1} = 2.$$

Now we compute

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} e^{\ln a_n} = e^2.$$

We conclude that

$$\boxed{\text{the sequence } \{a_n\} \text{ converges to } e^2.}$$