

PRINT Your Name: _____

Quiz 7 — October 2, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.

Circle your answer. **Show your work.** Check your answer!

The quiz is worth 5 points.

Compute $\int \frac{dx}{2x^2 + 4x + 7}$.

Answer: Complete the square: $2x^2 + 4x + 7 = 2(x^2 + 2x + \boxed{1}) + 7 - \boxed{2} = 2(x+1)^2 + 5$.

Let $\sqrt{2}(x+1) = \sqrt{5} \tan \theta$. It follows that $\sqrt{2}dx = \sqrt{5} \sec^2 \theta d\theta$. We compute

$$2(x+1)^2 + 5 = 5 \tan^2 \theta + 5 = 5(\tan^2 \theta + 1) = 5 \sec^2 \theta.$$

The integral is

$$\int \frac{\frac{\sqrt{5} \sec^2 \theta d\theta}{\sqrt{2}}}{5 \sec^2 \theta} = \frac{1}{\sqrt{10}} \int d\theta = \frac{1}{\sqrt{10}} \theta + C = \boxed{\frac{1}{\sqrt{10}} \arctan \left(\frac{\sqrt{2}(x+1)}{\sqrt{5}} \right) + C}.$$

Check. The derivative of the proposed answer is

$$\begin{aligned} & \left(\frac{1}{\sqrt{10}} \right) \left(\frac{\sqrt{2}}{\sqrt{5}} \right) \left(\frac{1}{1 + \left(\frac{\sqrt{2}(x+1)}{\sqrt{5}} \right)^2} \right) = \left(\frac{1}{5} \right) \left(\frac{1}{1 + \left(\frac{\sqrt{2}(x+1)}{\sqrt{5}} \right)^2} \right) \\ & = \left(\frac{1}{5 + (\sqrt{2}(x+1))^2} \right) = \left(\frac{1}{5 + (2(x^2 + 2x + 1))} \right) = \frac{1}{2x^2 + 4x + 7}. \checkmark \end{aligned}$$