

PRINT Your Name: \_\_\_\_\_

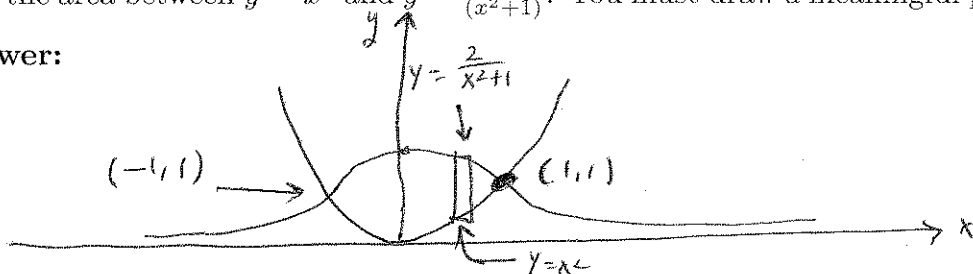
Quiz 6 — September 23, 2011 — Section 8 — 11:15 — 12:05

Remove everything from your desk except a pencil or pen.

The quiz is worth 5 points.

Find the area between  $y = x^2$  and  $y = \frac{2}{(x^2+1)}$ . You must draw a meaningful picture.

Answer:



The intersection occurs when  $y = \frac{2}{y+1}$ ; so  $y(y+1) = 2$  or  $y^2 + y - 2 = 0$  or  $(y+2)(y-1) = 0$ . Thus,  $y = -2$  or  $y = 1$ . Of course, if  $y = 1$ , then  $x$  is equal to 1 or  $-1$ . If  $y = -2$ , then there is no corresponding real value for  $x$ . Chop the  $x$ -axis from  $x$  equals  $-1$  to  $x = 1$ . Over each piece of the  $x$ -axis, we draw a rectangle of area  $h \cdot t$ , where  $h$  is the height and  $t$  is the thickness. We have  $t = \Delta x$ . We must express  $h$  in terms of  $x$ . We see that  $h$  is equal to the  $y$ -value at the top of the rectangle minus the  $y$ -value at the bottom of the rectangle. Thus,  $h = \frac{2}{(x^2+1)} - x^2$ . The area of the rectangle is  $h \cdot t = (\frac{2}{(x^2+1)} - x^2)\Delta x$ . The area of the region is

$$\int_{-1}^1 \left( \frac{2}{(x^2+1)} - x^2 \right) dx = \left( 2 \arctan x - \frac{x^3}{3} \right) \Big|_{-1}^1 = 2 \left( 2 \arctan(1) - \frac{1}{3} \right)$$
$$= \boxed{\pi - \frac{2}{3}}.$$