

PRINT Your Name: \_\_\_\_\_

**Quiz 5 — September 26, 2012 — Section 9 — 10:10 — 11:00**

**Remove everything from your desk except a pencil or pen.**

**Circle** your answer. **Show your work.** Your work should be correct and coherent. **CHECK** your answer.

The quiz is worth 5 points.

**Find**  $\int \frac{1}{x\sqrt{4x^2+1}} dx$ . **Check your answer.**

Let  $2x = \tan \theta$ . It follows that  $2dx = \sec^2 \theta d\theta$  and  $4x^2 + 1 = \tan^2 \theta + 1 = \sec^2 \theta$ . The integral is

$$\begin{aligned} \left(\frac{1}{2}\right) \int \frac{\sec^2 \theta d\theta}{\frac{\tan \theta}{2} \sec \theta} &= \int \frac{\sec^2 \theta d\theta}{\tan \theta \sec \theta} = \int \frac{\sec \theta d\theta}{\tan \theta} = \int \frac{\sec \theta \tan \theta d\theta}{\tan^2 \theta} \\ &= \int \frac{\sec \theta \tan \theta d\theta}{\sec^2 \theta - 1}. \end{aligned}$$

Let  $u = \sec \theta$ . It follows that  $du = \sec \theta \tan \theta d\theta$ . The integral is

$$\begin{aligned} &= \int \frac{du}{u^2 - 1} = \frac{1}{2} \int \left( \frac{1}{u-1} - \frac{1}{u+1} \right) du = \frac{1}{2} (\ln |u-1| - \ln |u+1|) \\ &= \frac{1}{2} (\ln |\sec \theta - 1| - \ln |\sec \theta + 1|) = \boxed{\frac{1}{2} (\ln |\sqrt{4x^2+1} - 1| - \ln |\sqrt{4x^2+1} + 1|)}. \end{aligned}$$

**Check.** The derivative of the proposed answer is

$$\begin{aligned} &\frac{1}{2} \left( \frac{\frac{8x}{2\sqrt{4x^2+1}}}{\sqrt{4x^2+1}-1} - \frac{\frac{8x}{2\sqrt{4x^2+1}}}{\sqrt{4x^2+1}+1} \right) \\ &= \frac{2x}{\sqrt{4x^2+1}} \left( \frac{1}{\sqrt{4x^2+1}-1} - \frac{1}{\sqrt{4x^2+1}+1} \right) \\ &= \frac{2x}{\sqrt{4x^2+1}} \left( \frac{\sqrt{4x^2+1}+1}{(\sqrt{4x^2+1}-1)(\sqrt{4x^2+1}+1)} - \frac{\sqrt{4x^2+1}-1}{(\sqrt{4x^2+1}+1)(\sqrt{4x^2+1}-1)} \right) \\ &= \frac{2x}{\sqrt{4x^2+1}} \left( \frac{2}{(4x^2+1)-1} \right) = \frac{2x}{\sqrt{4x^2+1}} \frac{2}{4x^2} = \frac{1}{\sqrt{4x^2+1}} \frac{1}{x}. \checkmark \end{aligned}$$