

PRINT Your Name: _____

Quiz 4 — September 15, 2010 — Section 10 — 11:15 — 12:05

Remove everything from your desk except this page and a pencil or pen.

Circle your answer. **Show your work.** **Check your answer.**

The quiz is worth 5 points.

Find $\int \frac{1}{(x+5)^2(x-1)} dx$.

Answer: Use the technique of partial fractions. Set

$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1}.$$

multiply both sides by $(x+5)^2(x-1)$ to obtain

$$\begin{aligned} 1 &= A(x+5)(x-1) + B(x-1) + C(x+5)^2 = A(x^2+4x-5) + Bx - B + C(x^2+10x+25) \\ &= (A+C)x^2 + (4A+B+10C)x + (-5A-B+25C). \end{aligned}$$

Equate the corresponding coefficients:

$$0 = A + C, \quad 0 = 4A + B + 10C, \quad 1 = -5A - B + 25C.$$

So, $C = -A$, $6A = -4A + 10A = -4A - 10C = B$, $1 = -5A - 6A - 25A$. We see that $1 = -36A$; that is $A = \frac{-1}{36}$, $B = \frac{-6}{36}$, and $C = \frac{1}{36}$. We check this much. We have

$$\begin{aligned} \frac{1}{36} \left[\frac{-1}{x+5} + \frac{-6}{(x+5)^2} + \frac{1}{x-1} \right] &= \frac{1}{36} \left[\frac{-(x+5)(x-1) - 6(x-1) + (x+5)^2}{(x+5)^2(x-1)} \right] \\ &= \frac{1}{36} \left[\frac{-(x^2+4x-5) - 6(x-1) + (x^2+10x+25)}{(x+5)^2(x-1)} \right] \\ &= \frac{1}{36} \left[\frac{-(-5) - 6(-1) + (+25)}{(x+5)^2(x-1)} \right] = \frac{1}{(x+5)^2(x-1)}, \end{aligned}$$

as expected. Now, we integrate

$$\begin{aligned} \int \frac{1}{(x+5)^2(x-1)} dx &= \frac{1}{36} \int \left[\frac{-1}{x+5} + \frac{-6}{(x+5)^2} + \frac{1}{x-1} \right] dx \\ &= \boxed{\frac{1}{36} \left[-\ln|x+5| + \frac{6}{(x+5)} + \ln|x-1| \right] + C}. \end{aligned}$$