

PRINT Your Name: \_\_\_\_\_

**Quiz 4 — February 7, 2014 – Section 8 – 10:50 – 11:40**

**Remove everything from your desk except this page and a pencil or pen.**

The solution will be posted soon after the quiz is given.

**Circle** your answer. **Show your work.** Your work must be correct and coherent. **Check your answer.**

The quiz is worth 5 points.

Find  $\int \sqrt{1-4x^2} dx$ .

**Answer:** We let  $2x = \sin \theta$ . It follows that  $2dx = \cos \theta d\theta$  and  $1 - 4x^2 = 1 - \sin^2 \theta = \cos^2 \theta$ . The original problem is

$$\begin{aligned} \int \sqrt{1-4x^2} dx &= \frac{1}{2} \int \cos^2 \theta d\theta = \frac{1}{4} \int (1 + \cos 2\theta) d\theta \\ &= \frac{1}{4}(\theta + \frac{1}{2} \sin 2\theta) + C = \frac{1}{4}(\theta + \sin \theta \cos \theta) + C \\ &= \frac{1}{4} (\arcsin (2x) + 2x\sqrt{1-4x^2}) + C = \boxed{\frac{1}{4} \arcsin (2x) + \frac{1}{2}x\sqrt{1-4x^2} + C}. \end{aligned}$$

Check. The derivative of the proposed answer is

$$\begin{aligned} \frac{1}{4} \frac{2}{\sqrt{1-4x^2}} + \frac{1}{2} \left[ x \frac{-8x}{2\sqrt{1-4x^2}} + \sqrt{1-4x^2} \right] &= \frac{1}{2} \left[ \frac{1}{\sqrt{1-4x^2}} - \frac{4x^2}{\sqrt{1-4x^2}} + \sqrt{1-4x^2} \right] \\ &= \frac{1}{2} \left[ \frac{1-4x^2}{\sqrt{1-4x^2}} + \sqrt{1-4x^2} \right] = \frac{1}{2} \left[ \sqrt{1-4x^2} + \sqrt{1-4x^2} \right] = \frac{1}{2} \left[ 2\sqrt{1-4x^2} \right] \\ &= \sqrt{1-4x^2}. \checkmark \end{aligned}$$