

PRINT Your Name: _____

Quiz 3 — September 2, 2011 — Section 7 — 10:10 — 11:00

Remove everything from your desk except a pencil or pen.

Circle your answer. **Show your work.** Your work should be correct and coherent. **CHECK** your answer.

The quiz is worth 5 points.

Find $\int \sqrt{5 + 4x - x^2} dx$.

Answer: Complete the square $5 + 4x - x^2 = 5 + \boxed{4} - (x^2 - 4x + \boxed{4}) = 9 - (x - 2)^2$. We let $x - 2 = 3 \sin \theta$. It follows that $dx = 3 \cos \theta d\theta$ and $9 - (x - 2)^2 = 9 - 9 \sin^2 \theta = 9 \cos^2 \theta$. The original problem is

$$\begin{aligned} \int \sqrt{5 + 4x - x^2} dx &= \int \sqrt{9 - (x - 2)^2} dx = 9 \int \cos^2 \theta d\theta = \frac{9}{2} \int (1 + \cos 2\theta) d\theta \\ &= \frac{9}{2} (\theta + (1/2) \sin 2\theta) + C = \frac{9}{2} (\theta + \sin \theta \cos \theta) + C \\ &= \frac{9}{2} \left(\arcsin \left(\frac{x - 2}{3} \right) + \frac{x - 2}{3} \frac{\sqrt{9 - (x - 2)^2}}{3} \right) + C \\ &= \frac{9}{2} \left(\arcsin \left(\frac{x - 2}{3} \right) + \frac{x - 2}{3} \frac{\sqrt{5 + 4x - x^2}}{3} \right) + C \\ &= \boxed{\frac{9}{2} \arcsin \left(\frac{x - 2}{3} \right) + \frac{1}{2} (x - 2) \sqrt{5 + 4x - x^2} + C} \end{aligned}$$

Check. The derivative of the proposed answer is

$$\begin{aligned} &\frac{9}{2} \frac{1/3}{\sqrt{1 - \left(\frac{x-2}{3}\right)^2}} + (1/2) \left[(x - 2) \frac{4 - 2x}{2\sqrt{5 + 4x - x^2}} + \sqrt{5 + 4x - x^2} \right] \\ &= \frac{9}{2} \frac{1/3}{\frac{1}{3} \sqrt{9 - (x - 2)^2}} + (1/2) \left[(x - 2) \frac{2 - x}{\sqrt{5 + 4x - x^2}} + \sqrt{5 + 4x - x^2} \right] \\ &= \frac{9}{2} \frac{1}{\sqrt{5 + 4x - x^2}} + (1/2) \left[(x - 2) \frac{2 - x}{\sqrt{5 + 4x - x^2}} + \sqrt{5 + 4x - x^2} \right] \\ &= \frac{1}{2\sqrt{5 + 4x - x^2}} [9 - (x - 2)^2 + 5 + 4x - x^2] \\ &= \frac{1}{2\sqrt{5 + 4x - x^2}} [2(5 + 4x - x^2)] = \sqrt{5 + 4x - x^2}. \checkmark \end{aligned}$$