

PRINT Your Name: _____

Quiz 3 — September 16, 2012 — Section 2 — 4:40 — 5:30

Remove everything from your desk except this page and a pencil or pen.

The solution will be posted soon after the quiz is given.

Circle your answer. **Show your work.** Your work must be correct and coherent. Check your answer.

The quiz is worth 5 points.

Find $\int \frac{1}{(x+5)^2(x-1)} dx$.

Answer: We apply the technique of partial fractions. We must find constants A , B , C so that

$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1}.$$

Multiply both sides by $(x+5)^2(x-1)$. We must solve

$$1 = A(x+5)(x-1) + B(x-1) + C(x+5)^2$$

$$1 = \begin{cases} Ax^2 & +4Ax & -5A \\ & +Bx & -B \\ +Cx^2 & +10Cx & +25C \end{cases}$$

We must find constants A , B , and C so that

$$1 = (A+C)x^2 + (4A+B+10C)x + (25C-5A-B).$$

Equate the corresponding coefficients. We must solve

$$\begin{aligned} 0 &= A + C \\ 0 &= 4A + B + 10C \\ 1 &= -5A - B + 25C \end{aligned}$$

The top equation gives $C = -A$. The middle equation becomes $0 = -6A + B$. The bottom equation becomes $1 = -30A - B$. Add the two most recent equations to see $1 = -36A$. Thus,

$$A = \frac{-1}{36}, \quad B = \frac{-6}{36}, \quad \text{and} \quad C = \frac{1}{36}.$$

We have shown that

$$\frac{1}{(x+5)^2(x-1)} = \frac{1}{36} \left(\frac{-1}{x+5} + \frac{-6}{(x+5)^2} + \frac{1}{x-1} \right).$$

By the way this is correct. The right hand side is

$$\begin{aligned} \frac{-(x+5)(x-1) - 6(x-1) + (x+5)^2}{36(x+5)^2(x-1)} &= \frac{-(x+5)(x-1) - 6(x-1) + (x+5)^2}{36(x+5)^2(x-1)} \\ &= \frac{1}{(x+5)^2(x-1)}. \end{aligned}$$

We now have

$$\begin{aligned} \int \frac{1}{(x+5)^2(x-1)} dx &= \frac{1}{36} \int \left(\frac{-1}{x+5} + \frac{-6}{(x+5)^2} + \frac{1}{x-1} \right) dx \\ &= \boxed{\frac{1}{36} \left(-\ln|x+5| + \frac{6}{x+5} + \ln|x-1| \right) + C.} \end{aligned}$$