PRINT Your Name:____________________

Quiz 28 — November 19, 2015

Approximate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ with an error at most 5×10^{-5}. Justify your answer.

We apply the alternating series test. We see that the series is alternating; $\frac{1}{n^5} > \frac{1}{(n+1)^5}$ (so the terms in absolute value are decreasing), and $\lim_{n\to\infty} \frac{1}{n^5} = 0$ (so the terms go to zero). The alternating series test applies and

$$\left| \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5} - \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n^5} \right| \leq \frac{1}{(N+1)^5}.$$

We want the error to be less than or equal to 5×10^{-5}; so we make N large enough so that $\frac{1}{(N+1)^5} \leq 5 \times 10^{-5}$. We make $10^5 \leq 5(N+1)^5$. This inequality certainly happens when $N = 9$. We conclude that

$$\boxed{\sum_{n=1}^{9} \frac{(-1)^{n+1}}{n^5} \text{ approximates } \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5} \text{ with an error at most } 5 \times 10^{-5}.}$$