PRINT Your Name:____________________

Quiz 27 — November 18, 2015

Does $\sum_{n=1}^{\infty}(1+\frac{1}{n})^2e^{-n}$ converge? Justify your answer.

We apply the Limit Comparison Test to the given series and $\sum_{n=1}^{\infty}e^{-n}$. Both series are series of positive numbers. We know that $\sum_{n=1}^{\infty}e^{-n}$ is the geometric series with initial term $\frac{1}{e}$ and ratio $\frac{1}{e}$. The number $\frac{1}{e}$ satisfies $-1<\frac{1}{e}<1$; so $\sum_{n=1}^{\infty}e^{-n}$ converges. We compute

$$\lim_{n\to\infty}\frac{(1+\frac{1}{n})^2e^{-n}}{e^{-n}}=\lim_{n\to\infty}(1+\tfrac{1}{n})^2=1.$$

Of course, 1 is a nonzero number; so the Limit Comparison Test ensures that $\sum_{n=1}^{\infty}(1+\frac{1}{n})^2e^{-n}$ and $\sum_{n=1}^{\infty}e^{-n}$ both converge or both diverge. We saw that $\sum_{n=1}^{\infty}e^{-n}$ converges; so

$$\boxed{\sum_{n=1}^{\infty}(1+\tfrac{1}{n})^2e^{-n}\text{ also converges.}}$$