PRINT Your Name:____________________

Quiz 25 — November 12, 2015

Does $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ converge? Justify your answer.

We compare the given series to the divergent harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$. We use the limit comparison test. Observe that

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}} = 1,$$

which is a non-zero finite number. The limit comparison test ensures that $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ both converge or both diverge. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges; thus the series

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}} \text{ also diverges.}}$$