PRINT Your Name:____________________

Quiz 16 — October 14, 2015

Find $\int \frac{3x^2-3x+2}{x^3+x}dx$**. Please check your answer.**

We use the technique of partial fractions. The demoninator factors as $x(x^2+1)$. We look for A, B, and C with

$$\frac{3x^2-3x+2}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}.$$

Multiply both sides by $x(x^2+1)$ to obtain

$$3x^2-3x+2 = A(x^2+1)+(Bx+C)x = (A+B)x^2+Cx+A.$$

Equate the corresponding coefficients to see that

$$3 = A+B, \quad -3 = C, \quad 2 = A.$$

Calculate that $B = 1$. We seem to have shown that

$$\frac{3x^2-3x+2}{x^3+x} = \frac{2}{x} + \frac{x-3}{x^2+1}.$$

We verify this claim before going any further. The right side is

$$\frac{2x^2+2+x^2-3x}{x(x^2+1)} = \frac{3x^2-3x+2}{x^3+x},$$

as claimed. Now we compute

$$\int \frac{3x^2-3x+2}{x^3+x}dx = \int \left(\frac{2}{x} + \frac{x-3}{x^2+1}\right) dx$$

$$= \boxed{2\ln x + \frac{1}{2}\ln(x^2+1) - 3\arctan x + C}.$$