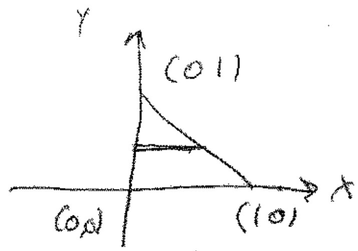
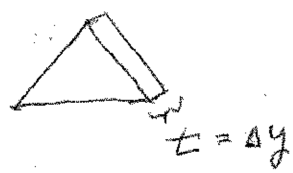
Quiz 15 September 30, 2015

Consider a solid whose base is the triangular region with vertices $(0,0)$, $(1,0)$, and $(0,1)$. The cross sections of this solid perpendicular to the y-axis are equilateral triangles. Find the volume of the solid.

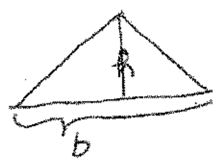
The base of the solid is



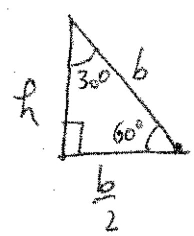
We partition the y-axis from $y = 0$ to $y = 1$. We consider the slice above each small interval of the y-axis



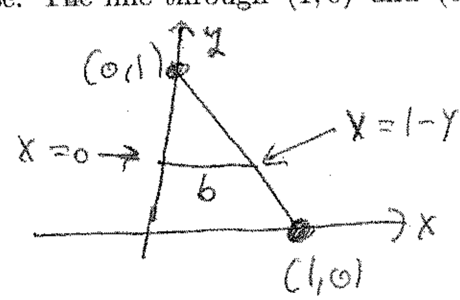
This slice has volume equal to the area of the face times the thickness. The volume of the slice is $\frac{1}{2}bht$ where b is the base of the triangle and h is the height of the triangle.



The triangle is equilateral so $h = \frac{\sqrt{3}}{2}b$.



The volume of the slice is $\frac{1}{2}bht = \frac{1}{2}\frac{\sqrt{3}}{2}b^2\Delta y$. We can express b in terms of y by looking at the original picture of the base. The line through $(1,0)$ and $(0,1)$ is $x + y = 1$. We see that $b = 1 - y$:



The volume of the slice is $= \frac{1}{2}\frac{\sqrt{3}}{2}b^2\Delta y = \frac{\sqrt{3}}{4}(1-y)^2\Delta y$. The volume of the solid is

$$\frac{\sqrt{3}}{4}\int_0^1 (1-y)^2 dy = -\frac{\sqrt{3}}{4}\frac{(1-y)^3}{3}\Bigg|_0^1 = \boxed{\frac{\sqrt{3}}{12}}.$$