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Quiz – March 23, 2006

Consider the series

$$e^{-x} + e^{-2x} + e^{-3x} + e^{-4x} + e^{-5x} + \dots$$

Find all values of x for which the series converges and find the sum of the series for those values of x . **Explain thoroughly.**

Answer: The series is a geometric series with initial term $a = e^{-x}$ and ratio $r = e^{-x}$. If $-1 < e^{-x} < 1$, then the series converges to

$$\frac{a}{1-r} = \frac{e^{-x}}{1-e^{-x}} = \frac{1}{e^x-1}.$$

Of course, e^{-x} is always positive; so, the only constraint is that $\frac{1}{e^x} < 1$. This is the same as saying $1 < e^x$, and this is the same as $0 < x$. We conclude that

if $0 < x$, then the series converges to $\frac{1}{e^x-1}$.
