PRINT Your Name:____________________

Quiz – March 2, 2006

Consider the sequence $\{a_n\}_{n=1}^{\infty}$, where

$$a_n = \frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{n}{n^2}.$$

What is the limit of this sequence? Explain your answer thoroughly.

Answer: Recall that if $S = 1 + 2 + 3 + \ldots n$, then $S = \frac{n(n+1)}{2}$. (Indeed, $2S$ is equal to

$$\begin{array}{llllll} 1 & +2 & +3 & +\ldots & +(n-1) & +n \\ +n & +(n-1) & +(n-2) & +\ldots & +1 & +n. \end{array}$$

Each column adds to $n+1$, there are n columns. Thus, $2S = n(n+1)$ and $S = \frac{n(n+1)}{2}$.) It follows that

$$a_n = \frac{n(n+1)}{2n^2} = \frac{n^2}{2n^2} + \frac{1}{2n};$$

and

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2} + \frac{1}{2n} = \boxed{\frac{1}{2}}.$$