Evaluate the indefinite integral $\int \frac{t}{1-t^8} dt$ as a power series. What is the radius of convergence?

Answer. The geometric series $\sum_{n=0}^{\infty} (t^8)^n$ converges to $\frac{1}{1-t^8}$ for $-1 < t^8 < 1$. Notice that $-1 < t^8 < 1$ if and only if $-1 < t < 1$. So

$$\sum_{n=0}^{\infty} t^{8n} = \frac{1}{1-t^8} \quad \text{for } -1 < t < 1.$$

Multiply by t to see that

$$\sum_{n=0}^{\infty} t^{8n+1} = \frac{t}{1-t^8} \quad \text{for } -1 < t < 1.$$

Integrate to see that

$$\boxed{\sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2} + C = \int \frac{t}{1-t^8} dt \quad \text{for } -1 < t < 1.}$$