PRINT Your Name:__________________

Quiz 13 — November 28, 2012 – Section 10 – 11:15 – 12:05

Remove everything from your desk except a pencil or pen.

Write in complete sentences.
The quiz is worth 5 points.

Does the series $\sum_{n=1}^{\infty} \frac{\sin(\frac{1}{n})}{\sqrt{n}}$ converge? **Justify your answer very thoroughly. Write in complete sentences.**

Answer. When n is large, $\sin(\frac{1}{n})$ behaves much like $\frac{1}{n}$. For that reason, the given series makes us think of $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$. The numbers $\frac{\sin(\frac{1}{n})}{\sqrt{n}}$ and $\frac{1}{n^{3/2}}$ are positive. We use the Limit Comparison Test to compare the given series $\sum_{n=1}^{\infty} \frac{\sin(\frac{1}{n})}{\sqrt{n}}$ and the series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ that we made up. (Of course, the series that we made up is the p-series with $p = \frac{3}{2} > 1$. The series that we made up converges.) We compute

$$\lim_{n\to\infty} \frac{\text{“}a_n\text{”}}{\text{“}b_n\text{”}} = \lim_{n\to\infty} \frac{\frac{\sin(\frac{1}{n})}{\sqrt{n}}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}}.$$

The top and the bottom both go to zero. We use L'Hopital's rule to see that this limit is

$$\lim_{n\to\infty} \frac{\cos(\frac{1}{n})(\frac{-1}{n^2})}{\frac{-1}{n^2}} = \lim_{n\to\infty} \cos(\tfrac{1}{n}) = 1.$$

The limit 1 is a number. One is not zero. One is not infinity. The Limit Comparison Test guarantees that the series $\sum_{n=1}^{\infty} \frac{\sin(\frac{1}{n})}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ both converge or both diverge. We have already seen that $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges. We conclude that

$$\boxed{\sum_{n=1}^{\infty} \frac{\sin(\frac{1}{n})}{\sqrt{n}} \text{ also converges.}}$$