PRINT Your Name:____________________

Quiz 13 — November 25, 2013 – Section 2 – 4:40 – 5:30

Remove everything from your desk except a pencil or pen.

Write in complete sentences. Explain your work!
The quiz is worth 5 points.

Express $f(x) = \frac{3}{x^2-x-2}$ as a power series by first using partial fractions. **Justify your answer. Write in complete sentences.**

Answer: Observe that $x^2 - x - 2 = (x-2)(x+1)$. Write $\frac{3}{x^2-x-2} = \frac{A}{x-2} + \frac{B}{x+1}$. Multiply both sides by $(x-2)(x+1)$ to get $3 = A(x+1) + B(x-2)$. Plug in $x = 2$ to see that $A = 1$. Plug in $x = -1$ to see that $B = -1$. Check that

$$\frac{1}{x-2} + \frac{-1}{x+1} = \frac{(x+1)-(x-2)}{(x-2)(x+1)} = \frac{3}{x^2-x+1}.$$

Divide top and bottom by -2 to see that

$$\frac{1}{x-2} = \frac{\frac{-1}{2}}{1-\frac{x}{2}} = \frac{-1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n = \sum_{n=0}^{\infty}\left(\frac{-1}{2^{n+1}}\right)x^n \quad \text{for } -1 < \frac{x}{2} < 1.$$

Observe also that

$$\frac{-1}{x+1} = \frac{-1}{1-(-x)} = -\sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty}(-1)^{n+1}x^n \quad \text{for } -1 < -x < 1.$$

We see that $-1 < \frac{x}{2} < 1$ is equivalent to $-2 < x < 2$. The x's for which $-2 < x < 2$ and $-1 < -x < 1$ both occur are the x's with $-1 < -x < 1$. We conclude that

$$\boxed{f(x) = \sum_{n=0}^{\infty}\left((-1)^{n+1} + \frac{-1}{2^{n+1}}\right)x^n \quad \text{for } -1 < -x < 1.}$$