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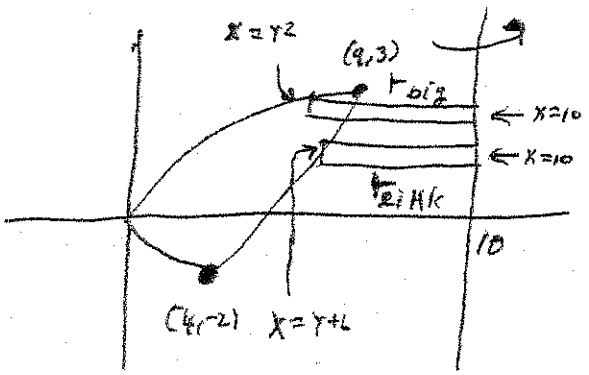
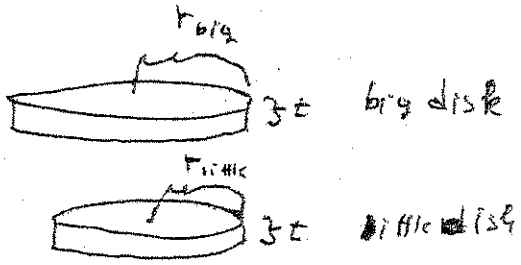
Quiz 12 — November 9, 2010 – Section 9 – 10:10 – 11:00

Consider the region bounded by $x = y^2$ and $y = x - 6$. Revolve the region about $x = 10$. Find the volume of the resulting solid.

We find the intersection points by solving $y = y^2 - 6$. This is $0 = y^2 - y - 6$, or $0 = (y-3)(y+2)$. So, $y = -2$ or $y = 3$. The intersection points are $(9, 3)$ and $(4, -2)$. We draw the parabola and the line. The line $x = 10$ is a vertical line to the right of the region. The easiest way to do the problem is by using big disks minus little disks. We partition the y-axis from $y = -2$ to $y = 3$ and we express everything in terms of y. In particular the thickness of each disk is $t = dy$.





The radius of the big disc is $10 - y^2$. The radius of the little disc is $10 - (y+6)$. The volume of the big disk is $\pi r^2 t = \pi(10 - y^2)^2 dy$. The volume of the small disk is $\pi r^2 t = \pi(4-y)^2 dy$. The volume of the solid is

$$\pi \int_{-2}^{3} [(10-y^2)^2 - (4-y)^2] dy = \pi \int_{-2}^{3} [100 - 20y^2 + y^4 - (16 - 8y + y^2)] dy$$

$$= \pi \int_{-2}^{3} [84 + 8y - 21y^2 + y^4] dy = \pi \left(\left(84y + 4y^2 - \frac{21}{3} y^3 + \frac{y^5}{5} \right) \Big|_{-2}^{3} \right)$$

$$= \boxed{\pi \left(84(3) + 4(9) - \frac{21}{3} 27 + \frac{3^5}{5} - \left(84(-2) + 4(4) - \frac{21}{3}(-2)^3 + \frac{(-2)^5}{5} \right) \right)}.$$