PRINT Your Name:____________________

Quiz 12 — September 23, 2015

Remove everything from your desk except this page and a pencil or pen. The solution will be posted soon after the quiz is given.

Circle your answer. **Show your work. Your work must be correct and coherent. Check your answer.**

Find $\int \frac{dx}{x+x\sqrt{x}}$.

Answer: Let $u = \sqrt{x}$. It follows that $du = \frac{dx}{2\sqrt{x}}$. In other words, $2udu = dx$. The original integral is

$$\int \frac{2u\,du}{u^2+u^3} = \int \frac{2du}{u+u^2} = \int \frac{2du}{u(1+u)}.$$

We use the technique of partial fractions and look for integers A and B with

$$\frac{2}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u}.$$

Multiply both sides by $u(1+u)$ to obtain

$$2 = A(1+u) + Bu.$$

Plug in $u = 0$ to learn that $A = 2$. Plug in $u = -1$ to learn that $B = -1$. Observe that

$$\frac{2}{u} - \frac{2}{u+1} = \frac{2u+2-2u}{u(u+1)} = \frac{2}{u(u+1)},$$

as desired. So, the original integral is

$$\int \left(\frac{2}{u} - \frac{2}{u+1}\right) du = 2\ln|u| - 2\ln|u+1| + C = 2\ln(\sqrt{x}) - 2\ln(\sqrt{x}+1) + C$$

$$= \boxed{\ln|x| - 2\ln(\sqrt{x}+1) + C}.$$

Check. The derivative of the proposed answer is

$$\frac{1}{x} - \frac{1}{\sqrt{x}(\sqrt{x}+1)} = \frac{x+\sqrt{x}-x}{x(x+\sqrt{x})} = \frac{\sqrt{x}}{x(x+\sqrt{x})} = \frac{1}{\sqrt{x}(x+\sqrt{x})} = \frac{1}{x\sqrt{x}+x}. \checkmark$$