PRINT Your Name:_____

Quiz 11 — April 4, 2014 – Section 8 – 10:50 – 11:40

Remove everything from your desk except this page and a pencil or pen. The solution will be posted soon after the quiz is given.

Circle your answer.
The quiz is worth 5 points.

Approximate the sum $\sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!}$ with an error at most 5×10^{-6}. Explain what you are doing **very thoroughly**. Your work must be correct and meaningful. **Write in complete sentences. Write from left to right and from top to bottom.**

Answer: The series is an alternating series. The absolute values of the terms form a decreasing sequence. (The numerators are constant and the denominators are growing.) The terms go to zero. The series converges by the Alternating Series Test and

$$\left| \sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!} - \sum_{n=0}^{N} \frac{(-1)^n}{10^n n!} \right| \leq \frac{1}{10^{N+1}(N+1)!}.$$

We look for N with

$$\frac{1}{10^{N+1}(N+1)!} \leq 5 \times 10^{-6}.$$

We look for N with

$$10^6 \leq 5 \cdot 10^{N+1}(N+1)!.$$

We notice that when $N = 3$, then

$$10^6 < 5 \cdot 10^4(24) = 5 \cdot 10^{N+1}(N+1)!.$$

We conclude that

$$\boxed{\sum_{n=0}^{3} \frac{(-1)^n}{10^n n!} \text{ approximates } \sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!} \text{ with an error at most } 5 \times 10^{-6}.}$$

Of course, our approximation is equal to $1 - \frac{1}{10} + \frac{1}{200} - \frac{1}{6000}$.