PRINT Your Name:_____

Remove everything from your desk except a pencil or pen.

Write in complete sentences.

The quiz is worth 5 points.

Does the series $\sum\limits_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ converge? Explain your work very thoroughly. Write in complete sentences.

Answer: Compare the given series to the convergent p-series $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$, where $p = 2 > 1$. We see that every term in both series $\sum\limits_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ and $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$ is positive. We also see that

$$\frac{\cos^2 n}{n^2 + 1} \leq \frac{1}{n^2 + 1}$$

because $\cos^2 n \leq 1$ and

$$\frac{1}{n^2 + 1} \leq \frac{1}{n^2}$$

because the denominator is larger on the left. Thus,

$$\frac{\cos^2 n}{n^2 + 1} \leq \frac{1}{n^2}.$$

The series $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$ converges; thus $\sum\limits_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$ also converges by the first part of the comparison test.