Quizzes 10 and 11 — April 6, 2011 – Section 3 – 8:00-8:50 recitation.

Each question is worth 5 points.

1. Does the series $\sum\limits_{n=2}^{\infty} \frac{\sqrt{n+2}}{2n^2+n+1}$ converge? **Justify your answer very thoroughly. Use complete sentences.**

2. Does the series $\sum\limits_{k=2}^{\infty} k^2e^{-k}$ converge? **Justify your answer very thoroughly. Use complete sentences.**

Answer. 1. Compare $\sum\limits_{n=2}^{\infty} \frac{\sqrt{n+2}}{2n^2+n+1}$ to $\sum\limits_{n=2}^{\infty} \frac{1}{n^{3/2}}$. The series $\sum\limits_{n=2}^{\infty} \frac{1}{n^{3/2}}$ is the p-series with $n = 3/2 > 1$. This series converges. We use the Limit Comparison. We observe that

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n+2}}{2n^2+n+1}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{(\sqrt{n+2})n^{3/2}}{2n^2+n+1} = \lim_{n\to\infty} \frac{\sqrt{1+\frac{2}{n}}}{2+\frac{1}{n}+\frac{1}{n^2}} = \frac{1}{2}.$$

We see that $1/2$ is a number; not 0 and not ∞. The Limit comparison test guarantees that $\sum\limits_{n=2}^{\infty} \frac{\sqrt{n+2}}{2n^2+n+1}$ and $\sum\limits_{n=2}^{\infty} \frac{1}{n^{3/2}}$ both converge or both diverge. We have seen that $\sum\limits_{n=2}^{\infty} \frac{1}{n^{3/2}}$ converges. We conclude that $\sum\limits_{n=2}^{\infty} \frac{\sqrt{n+2}}{2n^2+n+1}$ also converges.

2. Use the ratio test. We compute

$$\rho = \lim_{k\to\infty} \frac{(k+1)^2e^{-(k+1)}}{k^2e^{-k}} = \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^2 e^{-(k+1)}e^k = \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^2 e^{-1} = \frac{1}{e}.$$

Thus, $\rho < 1$. The ratio test guarantees that $\sum\limits_{k=2}^{\infty} k^2e^{-k}$ converges.